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GENERALIZED RHEOLOGICAL MODEL OF SHEAR DEFORMATION OF STRUCTURED CONDENSED MEDIA

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A relaxation-type macrorheological equation is constructed, which describes shear deformation of elastoplastoviscous media containing microvoids. The equation corresponds to regimes of steady-state creep and elastic and plastic strains, which pass to the regime of viscous flow with limited growth of voids at high shear strain rates.

Key words: *condensed media, shear deformation, viscous flow, microvoids.*

A typical feature of some solid–plastic materials, in addition to spallation fracture, is viscous fracture. One component of the latter process (in addition to the development of cavitation pores) is macroscale plastic deformation passing to the viscous flow regime under extremely high shear loads [1, 2]. Examples of such a process are flows in a shaped-charge jet and also in the region of formation of the contact surface in explosive welding of metals [3]. Moreover, viscous shear flow precedes the failure of the structure of liquid-plastic and liquid media whose properties lie in a wide range of rheological parameters [4–6]. Various aspects of dynamics of porous materials within the framework of elastoplastic model media have been studied in several papers [7–11]. It is also of interest to consider the problem of constructing a generalized model to describe the evolution of the structure of a condensed medium with microvoids (microbubbles in liquids and micropores in solid–plastic materials) in the entire range of shear deformation from creep to viscous flow. By definition, a rheological analog of such media is an elastoplastoviscous body (EPVB) [5] containing microvoids in its initial state.

1. Let us consider shear deformation of an EPVB containing microvoids. The mechanical analog of such a medium can be schematically presented as is shown in Fig. 1. Here G_0 is the modulus of shear elasticity of the EPVB, η_* is the coefficient of plastic viscosity of the EPVB for the St. Venant element (SV) passing through the yield point of the medium, μ_0 is the shear viscosity of the EPVB after the failure of its structure and the transition to the viscous (Newtonian) flow regime, μ_1 and G_1 are the viscous and elastic elements (parallel connection of G_1 and μ_1 forms the Voigt node corresponding to viscoelastic properties of the voids), and μ_2 is the shear viscosity of the medium in the creep regime.

The total strain of the EPVB can be presented as

$$\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_d + \varepsilon_c,$$

where ε_e and ε_p are the elastic and plastic strains of the medium, ε_d is the strain of disperse elements (voids), and ε_c is the creep-induced strains in the medium. Substituting the expressions relating the stress σ applied to the mechanical model and the strains in the nodes

$$\varepsilon_e = \frac{\sigma}{2\,G_0}, \qquad \varepsilon_p = \frac{\sigma}{2(\mu_0 + \eta_*)\,d/dt}, \qquad \varepsilon_d = \frac{\sigma}{2(G_1 + \mu_1\,d/dt)}, \qquad \varepsilon_c = \frac{\sigma}{2\mu_2\,d/dt}$$

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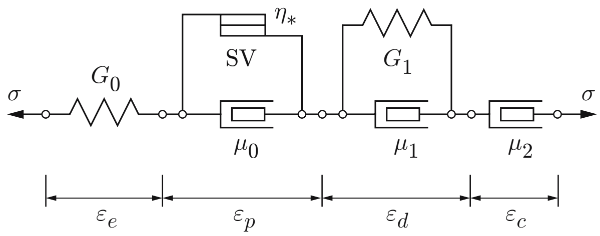

Fig. 1. Mechanical analog of an elastoplastoviscous medium containing microvoids.

into this equation, we obtain

$$(\mu_0+\eta_*)\mu_1\dddot{\sigma} + [G_0\mu_1 + (\mu_0+\eta_*)(G_0+G_1+G_0\mu_1/\mu_2)]\ddot{\sigma} + G_0G_1[1+(\mu_0+\eta_*/\mu_2)]\dot{\sigma}$$

$$= 2(\mu_0+\eta_*)G_0\mu_1\dddot{\varepsilon} + 2(\mu_0+\eta_*)G_0G_1\ddot{\varepsilon}. \qquad (1)$$

Integrating Eq. (1) with respect to time and passing to the three-dimensional case, we obtain

$$(\mu_0+\eta_*)\mu_1 T^{\oplus\oplus} + [G_0\mu_1 + (\mu_0+\eta_*)(G_0+G_1+G_0\mu_1/\mu_2)]T^{\oplus} + G_0G_1[1+(\mu_0+\eta_*)/\mu_2]T$$

$$= 2(\mu_0+\eta_*)G_0\mu_1 D^{\oplus} + 2(\mu_0+\eta_*)G_0G_1 D, \qquad (2)$$

where "$\oplus$" is the sign of the convective derivative; T and D are the tensor of additional viscous stresses (the meaning of this tensor is described below) and the strain-rate tensor, respectively. Multiplying Eq. (2) by T and taking into account that the relations $\eta_* \gg \mu_0$, $T\cdot T = 2\theta^2$ (this equality is valid when the yield point is reached), $T\cdot T^{\oplus} = 0$, and $T\cdot T^{\oplus\oplus} = -T^{\oplus}\cdot T^{\oplus}$ are satisfied in passing through the yield point, we find from the resultant equation multiplied by $(G_0G_1)^{-1}$ that

$$\eta_* = \frac{\theta^2}{\lambda_1 T^{\oplus}\cdot T^{\oplus}/(2G_0) + (D+\lambda_1 D^{\oplus})\cdot T - \theta^2/\mu_2}, \qquad (3)$$

where $\lambda_1 = \mu_1/G_1$. Finally, substituting Eq. (3) into Eq. (2) divided by G_0G_1 and performing certain transformations, we obtain a rheological equation for the EPVB containing voids:

$$\frac{\lambda_1}{G_0}\Big(\mu_0 + \frac{\theta^2}{D\cdot T + \lambda_1(D^{\oplus}\cdot T + T^{\oplus}\cdot T^{\oplus}/(2G_0)) - \theta^2/\mu_2}\Big)T^{\oplus\oplus}$$

$$+ \frac{1}{G_0}\Big[\mu_0 + \lambda_1 G_0 + \frac{(1+G_0/G_1)\theta^2}{D\cdot T + \lambda_1(D^{\oplus}\cdot T + T^{\oplus}\cdot T^{\oplus}/(2G_0)) - \theta^2/\mu_2} + \mu_0\,\frac{G_0}{G_1}$$

$$+ \lambda_1\,\frac{G_0}{\mu_2}\Big(\mu_0 + \frac{\theta^2}{D\cdot T + \lambda_1(D^{\oplus}\cdot T + T^{\oplus}\cdot T^{\oplus}/(2G_0)) - \theta^2/\mu_2}\Big)\Big]T^{\oplus}$$

$$+ \Big[1 + \frac{1}{\mu_2}\Big(\mu_0 + \frac{\theta^2}{D\cdot T + \lambda_1(D^{\oplus}\cdot T + T^{\oplus}\cdot T^{\oplus}/(2G_0)) - \theta^2/\mu_2}\Big)\Big]T$$

$$= 2\Big(\mu_0 + \frac{\theta^2}{D\cdot T + \lambda_1(D^{\oplus}\cdot T + T^{\oplus}\cdot T^{\oplus}/(2G_0)) - \theta^2/\mu_2}\Big)(D + \lambda_1 D^{\oplus}). \qquad (4)$$

Using notation (3), we can write this rheological equation in the form

$$\frac{\lambda_1}{G_0}\,(\mu_0+\eta_*)T^{\oplus\oplus} + \frac{1}{G_0}\Big[\mu_0 + \lambda_1 G_0 + \Big(1+\frac{G_0}{G_1}\Big)\eta_* + \mu_0\,\frac{G_0}{G_1} + \lambda_1\,\frac{G_0}{\mu_2}\,(\mu_0+\eta_*)\Big]T^{\oplus}$$

$$+ \Big(1 + \frac{\mu_0+\eta_*}{\mu_2}\Big)T = 2(\mu_0+\eta_*)(D+\lambda_1 D^{\oplus}).$$

In the case of pure shear, the rheological equation of the EPVB with voids acquires the form

$$\frac{\lambda_1}{G_0}(\mu_0+\eta_*)\ddot{\tau}+\frac{1}{G_0}\Big[\mu_0+\lambda_1 G_0+\Big(1+\frac{G_0}{G_1}\Big)\eta_*+\mu_0\,\frac{G_0}{G_1}+\lambda_1\,\frac{G_0}{\mu_2}(\mu_0+\eta_*)\Big]\dot{\tau}$$

$$+\Big(1+\frac{\mu_0+\eta_*}{\mu_2}\Big)\tau=2(\mu_0+\eta_*)(\dot{\varepsilon}+\lambda_1\ddot{\varepsilon}), \tag{5}$$

where

$$\eta_*=\frac{2\tau_*^2}{(\lambda_1/G_0)\dot{\tau}^2+2(\dot{\varepsilon}+\lambda_1\ddot{\varepsilon})\tau-2\tau_*^2/\mu_2}$$

and τ_* is the yield point under pure shear.

2. Let us analyze the void-containing EPVB behavior in various loading regimes.

2.1. The medium is subjected to a constant shear stress $\tau<\tau_*$ such that $\dot{\tau}=0$. As microvoids are not deformed thereby, we have $G_1\to\infty$, $\lambda_1\to 0$, and $G_0/G_1\ll 1$. In addition, the initial value of μ_2 remains almost unchanged because the medium structure is not destroyed or is changed only very weakly under these conditions. Hence, $\mu_2\gg\mu_0$ and $\eta_*\gg\mu_0$ in solid–plastic media, and Eq. (5) reduces to an equation of the form

$$\dot{\varepsilon}_c=f(\tau,t)=\frac{1}{2}\Big(\frac{1}{\eta_*}+\frac{1}{\mu_2}\Big)\tau, \tag{6}$$

where $\eta_*=\tau_*^2/(\dot{\varepsilon}\tau-\tau_*^2/\mu_2)$. From here, we can find the solution for steady-state creep in the simplest case with $\mu_2=\text{const}$:

$$\varepsilon_c=\varepsilon_c(0)+\frac{\tau}{2}\Big(\frac{1}{\mu_2}+\frac{1}{\eta_*}\Big)t. \tag{7}$$

Let us find the relation between Eqs. (6) and (7) and the known dependences of $\dot{\varepsilon}_c$ and ε_c on t for the creep regime in solid materials. In polymers, the strain consists of the elastic, viscoelastic, and viscoplastic components [12]. (The latter type of strains is caused by irreversible slipping of macromolecules with respect to each other at temperatures higher than the yield temperature.) The creep velocity asymptotically tends to a constant [2]: $\dot{\varepsilon}_c=A(\tau)$, i.e., the time evolution of the strain in the creep regime can be approximated by the relation

$$\varepsilon_c(t)=\varepsilon_c(0)+A(\tau)t. \tag{8}$$

For metals with a nonlinear dependence of $\dot{\varepsilon}_c$ on τ, we can use the flow theory

$$\dot{\varepsilon}_c=f(\tau), \tag{9}$$

if τ changes in time slowly and monotonically. Thus, Eqs. (6) and (7) derived in this work are similar to Eqs. (9) and (8), respectively, i.e., the generalized equation reduces to the known equations of steady-state creep [2, 12].

2.2. The medium is subjected to a shear stress varied in time, which is lower than the yield point, i.e., $\tau<\tau_*$ and $\dot{\tau}\neq 0$. In this case, we also have $G_1\to\infty$, $\lambda_1\to 0$, and $G_0/G_1\ll 1$. With allowance for this fact, we divide Eq. (5) by η_* to obtain

$$\frac{1}{G_0}\Big(1+\frac{\mu_0}{\eta_*}\Big)\dot{\tau}+\Big[\frac{1}{\eta_*}+\Big(1+\frac{\mu_0}{\eta_*}\Big)\frac{1}{\mu_2}\Big]\tau=2\Big(1+\frac{\mu_0}{\eta_*}\Big)\dot{\varepsilon}_e.$$

As $\mu_0/\eta_*\ll 1$, we have

$$\frac{1}{G_0}\,\dot{\tau}+\Big(\frac{1}{\eta_*}+\frac{1}{\mu_2}\Big)\tau=2\dot{\varepsilon}_e. \tag{10}$$

If τ is sufficiently small, so that $(1/\eta_*+1/\mu_2)\tau\ll\dot{\tau}/G_0$, then Eq. (10) reduces to Hooke's law $\tau=2G_0\varepsilon_e$ corresponding to elastic deformation of the medium. In the case of three-dimensional deformation with $T<\theta$, $T^{\oplus}\neq 0$, and $G_1\to\infty$, with allowance for $\mu_0/\mu_2\ll 1$, Eq. (4) reduces to an equation of the form

$$\Big(1+\frac{\theta^2}{\mu_2 D\cdot T-\theta^2}\Big)T+\frac{\mu_0}{G_0}\Big(1+\frac{1}{\mu_0}\,\frac{\theta^2}{D\cdot T-\theta^2/\mu_2}\Big)T^{\oplus}=2\Big(\mu_0+\frac{\theta^2}{D\cdot T-\theta^2/\mu_2}\Big)D \tag{11}$$

corresponding to the model of a homogeneous EPVB. If we assume that $\theta^2/\mu_2\ll 1$, Eq. (11) transforms to

$$T+\frac{1}{G_0}\Big(\mu_0+\frac{\theta^2}{D\cdot T}\Big)T^{\oplus}=2\Big(\mu_0+\frac{\theta^2}{D\cdot T}\Big)D \tag{12}$$

$[\mu_0 + \theta^2/(D \cdot T) = \mu_z$ is the structural viscosity of a homogeneous medium], which was validated experimentally [5]. At higher intensities of shear stresses $[\theta^2(\mu_0 D \times T)^{-1} \ll 1]$, because of the failure of the ordered structure of the medium ($\mu_z \to \mu_0$), Eq. (12) reduces to the well-studied Maxwell equation for Newtonian fluids [13]:

$$T + \lambda_0 T^{\oplus} = 2\mu_0 D, \qquad \lambda_0 = \mu_0/G_0. \tag{13}$$

2.3. If $T > \theta$ and $D \cdot T \gg \theta^2/\mu_2$, i.e., the deformation occurs in the high-velocity plastic flow regime and $(1 + G_0/G_1)\theta^2\{D \cdot T + \lambda_1[D^{\oplus} \cdot T + T \cdot T/(2G_0)] - \theta^2/\mu_0\}^{-1} \ll \mu_0$ thereby, with allowance for $\mu_0/\mu_2 \ll 1$, Eq. (4) yields

$$T + (\lambda_0 + \lambda_1 + \mu_0/G_1)T^{\oplus} + \lambda_0\lambda_1 T^{\oplus\oplus} = 2\mu_0(D + \lambda_1 D^{\oplus}). \tag{14}$$

If the loading regime is such that the term $\lambda_0\lambda_1 T^{\oplus\oplus}$ is very small, as compared with the first two terms in the left side of Eq. (14), this equation reduces to the classical Jeffries equation that describes the behavior of gels, emulsions, and suspensions (including gas-containing suspensions, i.e., liquids with bubbles) [14, 15]:

$$T + \tilde{\lambda} T^{\oplus} = 2\mu_0(D + \lambda_1 D^{\oplus}). \tag{15}$$

Here $\tilde{\lambda} = \lambda_0 + \lambda_1 + \mu_0/G_1$ and λ_1 is the time of retardation (retarded recovery of the shape of the deformed volume of the medium after the stress is removed). Obviously, retardation is caused by the presence of voids in the medium: as $G_1 \to \infty$, Eq. (15) reduces to the Maxwell equation (13).

Let us analyze the rheological characteristics of the EPVB containing voids by an example of the Couette flow for which the components of the velocity vector $\boldsymbol{v}$ in the Cartesian coordinate system (x^1, x^2, x^3) have the form $v^1 = \dot{\varepsilon}x^2$ and $v^2 = v^3 = 0$. We recall that the total stress tensor is

$$\chi = -pI + T = -pI + ((1/3)\operatorname{tr} T)I + S = -(p - (1/3)\operatorname{tr} T)I + S,$$

where p is the thermodynamic pressure, I is the unit tensor, T is the tensor of additional viscous stresses, $\operatorname{tr} T$ is the trace of the tensor T, and S is the deviator of the tensor T. In Eq. (15) written in matrix form, we substitute the values of the convective derivatives

$$T^{\oplus} = \frac{dT}{dt} + T \cdot W + (T \cdot W)^{\mathrm{T}} + a(T \cdot D + D \cdot T),$$

$$D^{\oplus} = \frac{dD}{dt} + D \cdot W + (D \cdot W)^{\mathrm{T}} + 2D^2,$$

where $D = (\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^{\mathrm{T}})/2$ and $W = (\nabla \boldsymbol{v} - \nabla \boldsymbol{v}^{\mathrm{T}})/2$. Taking into account that $dT/dt = 0$ and $dD/dt = 0$ for a steady-state shear flow, we decompose Eq. (15) with respect to the ith and jth components and obtain a system of algebraic equations where the components of the symmetric tensor of additional viscous stresses

$$T = \begin{pmatrix} \sigma^{11} & \tau^{12} & \tau^{13} \\ \tau^{21} & \sigma^{22} & \tau^{23} \\ \tau^{31} & \tau^{32} & \sigma^{33} \end{pmatrix}$$

have the form

$$\tau^{12} = \tau^{21} = \frac{\mu_0\dot{\varepsilon}_\tau + \lambda_1\tilde{\lambda}\mu_0\dot{\varepsilon}_\tau^3(1 - a^2)}{(1 - a^2)\tilde{\lambda}^2\dot{\varepsilon}_\tau^2 + 1}, \qquad \tau^{13} = \tau^{23} = \tau^{31} = \tau^{32} = 0,$$

$$\sigma^{11} = (1 - a)(\tilde{\lambda}\tau^{12} - \mu_0\lambda_1\dot{\varepsilon}_\tau)\dot{\varepsilon}_\tau, \qquad \sigma^{22} = (1 + a)(\lambda_1\mu_0\dot{\varepsilon}_\tau - \tilde{\lambda}\tau^{12})\dot{\varepsilon}_\tau, \qquad \sigma^{33} = 0$$

and, hence,

$$\tilde{\sigma} = \frac{1}{3}\operatorname{tr} T = \frac{1}{3}(\sigma^{11} + \sigma^{22} + \sigma^{33}) = -\frac{2}{3}a\mu_0\dot{\varepsilon}_\tau^2\frac{\tilde{\lambda} - \lambda_1}{1 + (1 - a^2)\tilde{\lambda}^2\dot{\varepsilon}_\tau^2}.$$

Thus, if there occurs a pure shear deformation (shear flow), i.e., $\dot{\varepsilon}_\tau \neq 0$, then $\mu_0 = 0$, $\tilde{\sigma} = 0$, and $\chi = -pI + S$ in an ideal liquid; in a viscous medium, however, $\tilde{\sigma} \neq 0$, i.e., stresses normal to the shear plane σ^{11} and σ^{22} are formed in the medium, which is validated experimentally (Weisenberg effect) [16]. The viscosimetric functions for the medium under study, i.e., the effective shear viscosity μ_* and the first N_1 and second N_2 differences of the normal stresses are written as

$$\mu_* = \frac{\tau^{12}}{\dot{\varepsilon}_\tau} = \mu_0 \, \frac{1 + (1 - a^2)\lambda_1 \dot{\varepsilon}_\tau \, \mathrm{De}}{1 + (1 - a^2)\, \mathrm{De}^2}; \tag{16}$$

$$N_1 = \sigma^{11} - \sigma^{22} = 2\mu_0 \, \frac{(\tilde{\lambda} - \lambda_1)\dot{\varepsilon}_\tau^2}{1 + (1 - a^2)\, \mathrm{De}^2} = 2\mu_0 \, \frac{(1 + G_0/G_1)\dot{\varepsilon}_\tau \, \mathrm{De}}{1 + (1 - a^2)\, \mathrm{De}^2}; \tag{17}$$

$$N_2 = \sigma^{22} - \sigma^{33} = -\frac{(1 + a)\mu_0(\tilde{\lambda} - \lambda_1)\dot{\varepsilon}_\tau^2}{1 + (1 - a^2)\, \mathrm{De}^2} = -\frac{(1 + a)(1 + G_0/G_1)\mu_0 \dot{\varepsilon}_\tau \, \mathrm{De}}{1 + (1 - a^2)\, \mathrm{De}^2}, \tag{18}$$

where $\mathrm{De} = \tilde{\lambda}\dot{\varepsilon}_\tau$ is the Debora number. As the effective viscosity has to be always positive, the parameter a should obey the inequality $-1 \leqslant a \leqslant 1$ in accordance with Eq. (16). It follows from [13] that the real processes in liquids are described only by the upper convective derivatives from the stress and strain-rate tensors, i.e., $-1 \leqslant a \leqslant 0$. Note, for a medium containing only practically undeformable micronuclei of voids, we have $G_1 \to \infty$, $\lambda_1 \to 0$, and functions (16)–(18) transform to a system of viscosimetric functions of the Maxwell model [5].

Let us analyze the dependence of the isotropic component of the total stress tensor $\chi = -(p - \tilde{\sigma})I + S$ on the shear strain rate $\dot{\varepsilon}_\tau$. As $-1 \leqslant a \leqslant 0$ and $\tilde{\lambda} = \lambda_0 + \lambda_1 + \mu_0/G_1$, the isotropic component of the tensor χ is written as

$$-\tilde{P}I = -(p - \tilde{\sigma})I = -\Big(p - \frac{2}{3}\, |a| \, \frac{(G_0 + G_1)\mu_0^2 \dot{\varepsilon}_\tau^2}{G_0 G_1[(1 - a^2)\tilde{\lambda}^2 \dot{\varepsilon}_\tau^2 + 1]}\Big) I,$$

i.e., for $\mu_0 \neq 0$ and $\dot{\varepsilon}_\tau \neq 0$, the hydrostatic pressure $\tilde{P}$ in the medium decreases, and a tensile stress is formed in the vicinity of microvoids. If this stress satisfies the condition of expansion of microvoids [17], their volume concentration increases. With a further increase in $\dot{\varepsilon}_\tau$, however, so that $(1 - a^2)\tilde{\lambda}^2 \dot{\varepsilon}_\tau^2 \gg 1$, we obtain

$$-\tilde{P}I \approx -\Big(p - \frac{2}{3}\, |a| \mu_0^2 \, \frac{G_0^{-1} + G_1^{-1}}{(1 - a^2)\tilde{\lambda}^2}\Big) I = -\Big(p - \frac{2}{3} \, \frac{|a|}{1 - a^2} \, G_1 \, \frac{1 + G_1/G_0}{(G_1/G_0 + \mu_1/\mu_0 + 1)^2}\Big) I.$$

As G_1 decreases with increasing voids, the condition $G_1/G_0 \ll 1$ is satisfied, and

$$\tilde{P} \to p - \frac{2}{3} \, \frac{|a|}{1 - a^2} \, \frac{G_1}{(1 + \mu_1/\mu_0)^2}.$$

Thus, with decreasing G_1, the hydrostatic pressure $\tilde{P}$ increases and tends to the thermodynamic ("initial") pressure p; the growth of voids is decelerated thereby.

As here we discuss an effect of principal importance, namely, the negative feedback in the mechanism of void growth (in the shear flow zone) restricting the evolution of cavitation, it seems reasonable to perform a comparative analysis of this process using another approach.

3. As the mechanical model shown schematically in Fig. 1 and the rheological equation (4) constructed on the basis of this model do not contain explicit dependences of the shear elasticity modulus G_1 and effective viscosity μ_* on the volume concentration of voids α, we use the following approach to constructing the rheological equation for the EPVB containing voids.

The expressions for the effective shear elasticity modulus G and effective viscosity of the medium containing voids with a fixed volume concentration α can be presented as [18]

$$G(\alpha) = \frac{(1 - \alpha)G_\infty}{1 + [(8 - 10\nu)/(7 - 5\nu)][1 + (1 - \alpha_m \alpha)/\alpha_m^2]\alpha}; \tag{19}$$

$$\mu = \mu_0/(1 - 1{,}09\sqrt[3]{\alpha}), \qquad \alpha < \alpha_m, \tag{20}$$

where G_∞ is the dynamic modulus of shear elasticity of a homogeneous matrix, $0.4 \leqslant \nu \leqslant 0.5$ is Poisson's ratio of the matrix, $\alpha_m \simeq 0.75$ is the volume concentration of the ultimate packing of voids, and μ_0 is the shear viscosity of the matrix. The mechanical model of a homogeneous medium rheologically equivalent to the examined medium containing voids with a fixed value $0 \leqslant \alpha < \alpha_m$ is shown in Fig. 2. Here $G(\alpha)$ and $\mu(\alpha)$ are the elasticity modulus and viscosity of the rheologically equivalent medium determined from (19) and (20), respectively; η_* and μ_c are the plastic and shear viscosities of the rheologically equivalent medium in the creep regime.

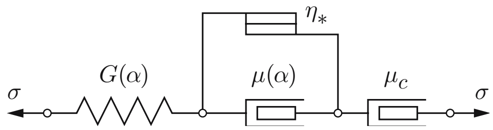

Fig. 2. Mechanical analog of a homogeneous medium rheologically equivalent to an elastoplastoviscous medium containing voids.

Using the method of constructing Eq. (4), based on the mechanical model of a rheologically equivalent medium, we obtain

$$\frac{\mu(\alpha)+\eta_*}{G(\alpha)}\,T^{\otimes}+\Big(1+\frac{\mu(\alpha)+\eta_*}{\mu_c}\Big)T=2[\mu(\alpha)+\eta_*]D, \tag{21}$$

where $\eta_* = \theta^2/(D\cdot T-\theta^2/\mu_c)$. For $T<\theta$ and $T^{\oplus}=0$, the voids are micronuclei ($\mu|_{\alpha\simeq 0}\simeq\mu_0$). With allowance for $\mu_0\ll\eta_*$, Eq. (21) yields

$$(1+\eta_*/\mu_c)T=2\eta_* D.$$

For pure shear, this equation yields the relations for steady-state creep

$$\dot\varepsilon_c=\frac{1}{2}\Big(\frac{1}{\eta_*}+\frac{1}{\mu_c}\Big)\tau, \qquad \varepsilon_c=\varepsilon_c(0)+\frac{\tau}{2}\Big(\frac{1}{\mu_c}+\frac{1}{\eta_*}\Big)t$$

similar to Eqs. (6) and (7), which coincide with the known equations for steady-state creep [2, 12].

In the case $T>\theta$ and $D\cdot T\gg\theta^2/\mu_c$, i.e., in the regime of high-velocity plastic deformation with the medium passing to a state with a destroyed structure, with allowance for $\mu(\alpha)\ll\eta_*$, Eq. (21) reduces to an equation of the form

$$\Big(1+\frac{\theta^2}{\mu_c D\cdot T}\Big)T+\frac{1}{G}\Big(\mu+\frac{\theta^2}{D\cdot T}\Big)T^{\oplus}=2\Big(\mu+\frac{\theta^2}{D\cdot T}\Big)D.$$

For $D\cdot T\gg\theta^2$, this equation reduces to the known Maxwell equation [13]

$$T+\lambda T^{\oplus}=2\mu D, \qquad \lambda=\mu/G.$$

Based on the model used in Sec. 2, the last equation yields the following relations for the Couette flow:

$$\mu_*=\frac{\mu}{1+(1-a^2)\,\mathrm{De}}, \qquad \sigma^{11}=\frac{(1-a)\mu\lambda\dot\varepsilon_\tau^2}{1+(1-a^2)\lambda^2\dot\varepsilon_\tau^2}, \qquad \sigma^{22}=-\frac{(1+a)\mu\lambda\dot\varepsilon_\tau^2}{1+(1-a^2)\lambda^2\dot\varepsilon_\tau^2},$$

$$\sigma^{33}=0, \qquad \tilde\sigma=\frac{1}{3}\,\mathrm{tr}\,T=\frac{2|a|}{3}\,\mu^2\,\frac{\dot\varepsilon_\tau^2}{G[1+(1-a^2)\lambda^2\dot\varepsilon_\tau^2]}, \tag{22}$$

$$N_1=\sigma^{11}-\sigma^{22}=\frac{2\mu\dot\varepsilon_\tau\,\mathrm{De}}{1+(1-a^2)\,\mathrm{De}^2}, \qquad N_2=\sigma^{22}-\sigma^{33}=-\frac{(1+a)\mu\dot\varepsilon_\tau\,\mathrm{De}}{1+(1-a^2)\,\mathrm{De}^2}, \qquad \mathrm{De}=\lambda\dot\varepsilon_\tau.$$

An analysis of the behavior of these functions depending on $\dot\varepsilon_\tau$ and α revealed the following. With increasing $\dot\varepsilon_\tau$, the value of μ_* first decreases and the values of N_1, N_2, and N_1-N_2 increase. With increasing $\dot\varepsilon_\tau$, the relation $(1-a^2)\,\mathrm{De}\gg 1$ being valid, the viscosimetric functions tend to the asymptotic expressions

$$\mu_*\to\frac{G(\alpha)}{(1-a^2)\dot\varepsilon_\tau}, \qquad N_1\to\frac{2G(\alpha)}{1-a^2}, \qquad N_2\to-\frac{G(\alpha)}{1-a}, \qquad N_1-N_2\to\frac{3+a}{1-a^2}\,G(\alpha),$$

i.e., like $G(\alpha)$, these functions monotonically decrease with increasing α.

Let us analyze the dependence of the hydrostatic pressure $\tilde P$ and, hence, the tensile stress in the medium, on $\dot\varepsilon_\tau$. The isotropic component of the total stress tensor $\chi=-pI+T=-(p-\tilde\sigma)I+S$ has the form

$$-\tilde P I=-(p-\tilde\sigma)I=-\Big(p-\frac{2}{3}\,|a|\,\frac{\mu\dot\varepsilon_\tau}{G[1+(1-a^2)\lambda^2\dot\varepsilon_\tau^2]}\Big)I,$$

i.e., for $\mu\neq 0$ and $\dot\varepsilon_\tau\neq 0$, the resultant hydrostatic pressure $\tilde P$ in the medium decreases. For this reason, a field of tensile stresses is formed in the vicinity of microvoids, which can lead to an increase in the size of microvoids if the energy inequality obtained in [17] is valid. With a further increase in $\dot{\varepsilon}_\tau$, the condition $(1-a^2)\lambda^2\dot{\varepsilon}_\tau^2 \gg 1$ being valid, formula (22) reduces to the expression

$$\tilde{\sigma} = \frac{2|a|}{3(1-a^2)}\, G(\alpha),$$

i.e., with allowance for Eq. (19), $\tilde{\sigma}$ is a decreasing function of α; thereby the pressure $\tilde{P}$ is recovered to the initial level p, thus, restricting the growth of voids. The analysis of the rheological equation (21) also implies the presence of negative feedback in the mechanism of void growth, which does not allow unlimited cavitation in the case of shear deformation of the EPVB containing microvoids at the initial time.

Thus, the rheological equation (4) is consistent with the generalized model of shear deformation of condensed (liquid, liquid-form, and solid–plastic) media containing microvoids. In all deformation regimes (steady-state creep, elastic deformation, plastic deformation, and viscous creep), the generalized model transforms to a series of known and experimentally validated models corresponding to these regimes of shear deformation of condensed media.

REFERENCES

1. H. Liebowitz (ed.), *Fracture*, Vol. 1: *Microscopic and Macroscopic Fundamentals*, Academic Press, New York–London (1968).
2. V. V. Rybin, *High Plastic Strains and Fracture of Metals* [in Russian], Metallurgiya, Moscow (1986).
3. M. A. Lavrent'ev and B. V. Shabat, *Hydrodynamics Problems and Their Mathematical Models* [in Russian], Nauka, Moscow (1977).
4. S. V. Stebnovskii, "Shear strength in structured water," *Zh. Tekh. Fiz.*, **74**, No. 1, 21–23 (2004).
5. S. V. Stebnovskii, "Tangential discontinuities of parameters of a polar fluid under shear strain," *J. Appl. Mech. Tech. Phys.*, **46**, No. 3, 339–345 (2005).
6. S. V. Stebnovskii, "Shear instability of the structure of media possessing viscous fluidity," *J. Appl. Mech. Tech. Phys.*, **47**, No. 2, 208–213 (2006).
7. V. Tvergaard, "Influence of voids on shear band instabilities under plane strain conditions," *Int. J. Fracture*, **17**, No. 4, 389–407 (1981).
8. E. I. Romenskii, "Relaxation model for describing the strain of porous materials," *J. Appl. Mech. Tech. Phys.*, **29**, No. 5, 735–738 (1988).
9. L. A. Merzhievskii and A. D. Resnyanskii, "Modeling of dynamic deformation of spheroplastic," *Fiz. Goreniya Vzryva*, **28**, No. 3, 119–121 (1992).
10. S. P. Kiselev and V. M. Fomin, "Model of a porous material considering the plastic zone near the pore," *J. Appl. Mech. Tech. Phys.*, **34**, No. 6, 861–869 (1993).
11. S. V. Stebnovskii, "Conditions for development of cavitation in scleronomous media," *J. Appl. Mech. Tech. Phys.*, **42**, No. 1, 77–86 (2001).
12. G. M. Bartenev and Yu. V. Zelenov, *Physics and Mechanics of Polymers* [in Russian], Vysshaya Shkola, Moscow (1983).
13. G. Astarita and G. Marrucci, *Principles of Non-Newtonian Fluid Mechanics*, McGraw-Hill, New York (1974).
14. M. Reiner, *Rheology*, Springer Verlag, Berlin (1958).
15. L. S. Artyushkov, *Dynamics of Non-Newtonian Fluids* [in Russian], Izd. St.-Peterb. Gos. Mor. Tekh. Univ., St. Petersburg (1997).
16. R. B. Bird and C. Curtiss, "Amazing polymer fluids," in: *Physics Abroad–86, Ser. A: Research* (collected scientific papers) [Russian translation], Mir, Moscow (1986), pp. 29–51.
17. S. V. Stebnovskii, "Generalized rheological model of cavitating condensed media," *J. Appl. Mech. Tech. Phys.*, **42**, No. 3, 482–492 (2001).
18. S. V. Stebnovskii, "Shear elasticity of liquid media containing bubbles," *Fiz. Goreniya Vzryva*, **26**, No. 3, 127–128 (1990).